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THE FLYING VEHICLE OPTIMUM FLIGHT TRAJECTORY DESIGN

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RINGKASAN : *Satu prosedur rekabentuk telah dicadangkan untuk perpindahan kenderaan terbang yang optima dari keadaan mula dan akhir pada satu satah, di bawah pengaruh medan graviti pusat di dalam ruang kosong dan di bawah pengaruh tujahan yang tetap. Kajian ini mengambil kira algoritma kawalan kenderaan terbang yang dibina menggunakan Teori Gunaan Kawalan Kualiti Optimum dan Program Linear Melalui Sudut Balingan. Perbandingan antara kedua-dua algoritma kawalan untuk kenderaan terbang telah dijalankan.*

ABSTRACT : A design procedure is suggested for the flying vehicle optimal transfer from the initial to a final state in one plane, in the central gravitation field, in empty space and under the action of fixed thrust. The given study considers the flying vehicle motion control algorithms built proceeding from the optimal control applied theory and using a linear programme by the angle of pitch. The comparison of these flying vehicle motion control algorithms was carried out.

KEYWORDS : Equations of flying vehicle motion, flying vehicle, optimal control

INTRODUCTION

Optimal control design, giving the best possible system of a particular type, is one of the most classical subjects within the wide field of control theory. Many important contributions to the area of optimal control, such as the maximum principle (Gamkrelidze, 1999), were given in the early 1960s. In the last few years, several books on optimal control appeared, and among them the book by Bryson, A. and Ho You-Chi (1969). During this period, the newly developed optimal control theory was often used in space applications. This investigation distinguishes the flying vehicle motion control algorithm built proceeding from the optimal control applied theory (Bryson & Ho You-Chi, 1969).

METHODOLOGY

A calculation procedure is suggested for the flying vehicle optimal transfer from the initial to a final state under the action of fixed thrust. The final state is characterized by setting the flying vehicle radius-vector magnitude relative to a centre of attraction, its speed and angle of velocity inclination to a local horizon. The following assumptions are proposed: (i) the Earth is ball-shaped and, during a transition phase, it does not turn; (ii) the Earth gravitation field is central; (iii) the flight of the flying vehicle occurs at the high altitude where aerodynamic resistance can be neglected; (iv) the flying vehicle motion is in one plane and (v) the flying vehicle control system is inertialess. The equations of flying vehicle motion in its centric rectangular and vertical-wind-body coordinate system (Gorbatenko *et.al.*, 1969) are as follows:

$$\overset{\circ}{\bar{x}} = \bar{f}(\bar{x},u,t). \tag{1}$$

$$\left.\begin{aligned}
\frac{dV}{dt} &= \frac{P}{m}\cos\alpha - g_0\left(\frac{R_0}{R}\right)^2 \sin\theta;\\
\frac{d\theta}{dt} &= \frac{P}{mV}\sin\alpha - g_0\left(\frac{R_0}{R}\right)^2 \frac{\cos\theta}{V} + \frac{V}{R}\cos\theta;\\
\frac{dR}{dt} &= V\sin\theta;\\
\frac{d\varphi}{dt} &= \frac{V}{R}\cos\theta;\\
\frac{dm}{dt} &= -\overset{\circ}{m}{}^{a};
\end{aligned}\right\} \tag{2}$$

m^{a}

where $\bar{x} = (V, \theta, R, \varphi, m)^T$ is the flying vehicle current state vector, $\bar{f}(\bar{x}, u, t)$ is the right side vector of differential equation system of flying vehicle motion, t is the current time of the flight, V is the flying vehicle velocity, θ is the angle of velocity vector to a local horizon, R is the magnitude of the flying vehicle radius-vector relative to a centre of attraction, φ is a geocentric latitude, m is the flying vehicle mass, P is the flying vehicle thrust level value, g_0 is the amount of acceleration of free fall on Earth's surface, R'_0 is the Earth radius, $\overset{o}{m}^a$ is the absolute value of the flying vehicle mass flow rate, α is the angle of attack.

The angle of attack is used as a guidance factor for the flying vehicle motion control:

$$u(t) = \alpha(t). \tag{3}$$

Given is the flying vehicle initial state:

$$\bar{x}(t^1_0), \quad \text{(five initial conditions)} \tag{4}$$

The constraints are imposed on the flying vehicle terminal state:

$$\bar{\psi} = [\bar{x}(t_f), t_f] = 0, \tag{5}$$

where

t_f - current time at the end of flight,

$\bar{\psi} = (\psi_1, \psi_2, \psi_3)^T$, (three conditions),

(i) $\psi_1 = V - V_f$ - by the velocity vector value at the end of the transition phase, (ii) $\psi_2 = \theta - \theta_f$ – by the angle of inclination of the velocity vector to a local horizon at the end of the transition phase, (iii) $\psi_3 = R - R_f$ - by the value of the flying vehicle radius-vector relative to the attraction centre at the end of the transition phase, where R_f is the magnitude of the flying vehicle radius-vector relative to a centre of attraction at the end of flight.

For a quality criterion we take a negative value of the attraction final mass, i.e.:

$$-m(t_f). \tag{6}$$

So, a challenge is to define such a control $u(t)$, which would provide a minimum amount of final mass deceleration $[-m(t_f)]$ with some differential constraints (1) and constraints on the terminal state (5).

The Hamiltonian function H (Bryson & Ho You-Chi, 1969) for this problem can be written as:

$$H(\bar{x},u,\bar{\lambda},t) = \lambda_1 \frac{dV}{dt} + \lambda_2 \frac{d\theta}{dt} + \lambda_3 \frac{dR}{dt} + \lambda_4 \frac{d\varphi}{dt} + \lambda_5 \frac{dm}{dt}, \tag{7}$$

where $\bar{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)^{T}$ - the Lagrange multiplier vector (weighing kernel).
The Euler-Lagrange set of equations looks like:

$$\overset{\circ}{\lambda} = -\left(\frac{\partial H}{\partial \bar{x}}\right)^{T}, \qquad \text{(5 equations)}, \tag{8}$$

$$\frac{\partial H}{\partial u} = 0, \qquad \text{(1 equation)}, \tag{9}$$

Terminal conditions yield the equalities (Bryson & Ho You-Chi, 1969) :

$$\bar{\lambda}^{T}(t_f) = \left(\frac{\partial \Phi}{\partial \bar{x}}\right)_{t=t_f}, \qquad \text{(5 conditions)}, \tag{10}$$

where

$$\Phi(\bar{x}, \bar{v}, t) = -m(t) + \bar{v}^{T} \psi(\bar{x}, t)$$
$$\bar{v} = (v_1, v_2, v_3)^{T}, \tag{11}$$

$$\Omega[\bar{x}, u, \bar{v}, t]_{t=t_f} \equiv \left(\frac{d\Phi}{dt}\right)_{t=t_f} = 0, \qquad \text{(1 condition)}, \tag{12}$$

The transition time termination is defined implicitly through terminal boundary conditions (10). Thus, we have to find a solution (Bryson & Ho You-Chi, 1969) of the system of 10 differential Equations (1) (set of equations of flying vehicle motion) and (8) (set of equations of costate variables) and define 4 values of unknown parameters $\bar{v}$ and t_f so that to meet five initial conditions (4) and nine terminal environment (5), (10), (12).

Here the definition of $u(t)$ is found using equation (9):

$$\alpha_1 = \operatorname{arctg}\left(\frac{\lambda_2}{\lambda_1 V}\right), \; \alpha_2 = \alpha_1 + \pi \tag{13}$$

By the time optimal control, $\hat{u}$ should minimize $H(u)$ (Bryson & Ho You-Chi, 1969) :

$$\hat{H}(\bar{x},\hat{u},\bar{\lambda},t) \leq H(\bar{x},u,\bar{\lambda},t).$$

Out of two values of the angle of attack α_1 and α_2 we choose the one that provides the minimum value of *H*. Solution of the problems (1)-(12) can be reduced to a solution of the six-parameter boundary-value problem (Smirnov & Nesterov, 1980) which, in turn, can be derived to a finding of roots of the set of equations:

$$V_0 = V(\bar{\kappa}),\ \theta_0 = \theta(\bar{\kappa}),\ R_0 = R(\bar{\kappa}),\ \varphi_0 = \varphi(\bar{\kappa}),\ m_0 = m(\bar{\kappa}),\ \Omega_g = \Omega(\bar{\kappa}), \quad (14)$$

where

$\bar{\kappa} = (t_f, v_1, v_2, v_3, \varphi_f, m_f)$ - the unknown parameter vector. (15)

Here : V_0, θ_0, R_0, φ_0, m_0 - the flying vehicle given parameters to an instant of time t_0, which should be counted in solving the boundary-value problem, Ω_g - from the condition (12).

Integration of the system of differential Equations (1) and (8) is made with a reverse pitch until t_0. Then we determine the residuals by parameters (14). Solution of the boundary-value problem goes on until the conditions (14) are met with the given accuracy. To solve the problem we make use of the Newton method for solution of the system of nonlinear equations.

In the case that the initial approximations of vector of the Lagrange factor $\bar{\lambda}(t_0)$ is sufficiently appropriate, a solution of the problems (1)-(12) can be reduced to a finding of three roots in the system of equations:

$$\begin{aligned} \theta_f &= \theta(\lambda_1(t_0),\lambda_2(t_0),\lambda_3(t_0)); \\ R_f &= R(\lambda_1(t_0),\lambda_2(t_0),\lambda_3(t_0)); \\ \Omega_g &= \Omega(\lambda_1(t_0),\lambda_2(t_0),\lambda_3(t_0)); \end{aligned} \quad (16)$$

where θ_f and R_f are the given values of flying vehicle parameters to an instant of time t_r, which are to be fulfilled by virtue of solution of a boundary-value problem, Ω_g is from the condition (12),

$\lambda_1(t_0)$, $\lambda_2(t_0)$, $\lambda_3(t_0)$ are the unknown parameters. (17)

Integration of a set of differential Equations (1) and (8) is made with a positive pitch from an instant of time t_0 until a fulfilment of the given condition by velocity at the end of transit $V = V_f$ is achieved, only then we determine the residuals by conditions (16). $V = V_f$ is the fourth condition added to conditions (16). So, in this case we can solve a four-parametric boundary-value problem.

RESULTS

The given study considers the flying vehicle motion control algorithms built proceeding from the optimal-control applied theory and using a linear programme by the angle of pitch $\vartheta = \vartheta_0 + \mathring{\vartheta} t$, where ϑ_0 is the initial value of the angle of pitch, and $\mathring{\vartheta}_0$ is the pitch-around manoeuvre angle speed.

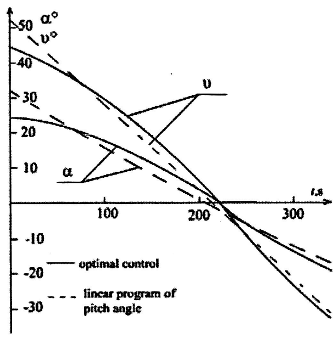

Figure 1. *The angle of attack and pitch angle variation*

Figure 1 illustrates the angle of attack and pitch angle variation dependences as to the flying vehicle flight time at optimal incidence control and with a linear programme of pitch control.

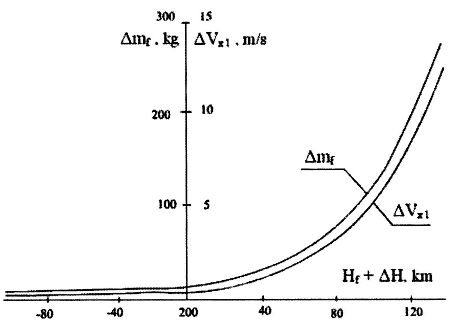

Figure 2. *Value variation of additional costs of the characteristic velocity showing the dependence on the altitude of the flying vehicle at the end of flight, ΔH_f*

Figure 2 illustrates a value variation of additional costs of the characteristic velocity ($\Delta V_{x1} = V_{x1\ lin} - V_{x1\ opt}$) with a pitch angle linear programme control relative to the costs wanted of the characteristic velocity under optimal incidence control and a residue change ($\Delta m_f = m_{f\ opt} - m_{f\ lin}$) between the flying vehicle final mass under optimal incidence control and that with a pitch linear programme depending on the altitude of the flying vehicle at the end of flight ΔH_f.

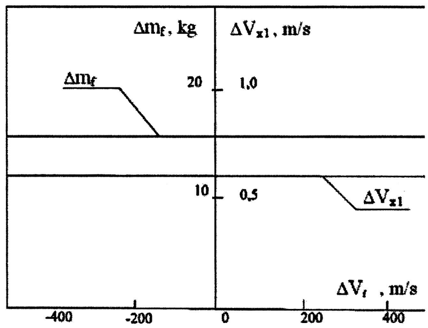

Figure 3. *Value variation of additional costs of the characteristic velocity showing the dependence on the velocity of the flying vehicle at the end of flight,* ΔV_f

Figure 3 illustrates a value variation of additional costs of the characteristic velocity with a pitch angle linear programme control relative to the costs wanted of the characteristic velocity under optimal incidence control and a residue change between the flying vehicle final mass under optimal incidence control and that with a pitch linear programme depending on the velocity of the flying vehicle at the end of flight ΔV_f.

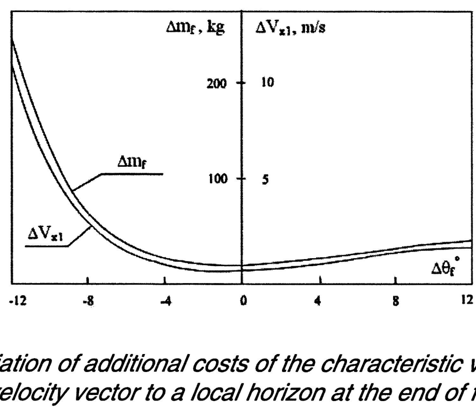

Figure 4. *Value variation of additional costs of the characteristic velocity showing the dependence of the velocity vector to a local horizon at the end of flight,* $\Delta\theta_f$

Figure 4 illustrates a value variation of additional costs of the characteristic velocity with a pitch angle linear programme control relative to the costs wanted of the characteristic velocity under optimal incidence control and a residue change between the flying vehicle final mass under optimal incidence control and that with a pitch linear programme depending on the angle of velocity vector to a local horizon at the end of flight $\Delta\theta_f$.

CONCLUSION

The analysis of dependences described herein proves that from the energy point of view the flying vehicle motion control algorithm based on the optimal control applied theory is more profitable as compared with linear program by angle of pitch.

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